

## Analysis on Phase Center Error of Star-Shaped GNSS Antenna without Jamming

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**Abstract:** In order to enable high-precision positioning of satellite navigation receivers using GNSS array antennas, it is necessary to select an appropriate antenna array according to requirements and consider the error of the phase center of the antenna array. Under the condition of no interference, the antenna array and its phase center error model are established by taking the common star-shaped antenna array as an example. The phase center error of different satellites with different array elements is analyzed and compared. The simulation results show that the phase center error of the antenna array has a periodicity when the satellite signal changes along the azimuth. When there is no interference, the phase center of the star-shaped antenna array composed of odd array elements coincides with its geometric center, and the phase error is much smaller than that of the even array elements. The array can meet the high-precision positioning requirements without considering the factors such as mutual coupling of array elements.

### 1. Introduction

The phase center generally refers to the center of a sphere with equal phase of electromagnetic radiation field [1, 2]. In satellite navigation receivers, the antenna phase center error is an error source that cannot be ignored in the positioning solution. When the receiver uses a single-element antenna, the phase center is relatively fixed, resulting in a small measurement error, usually in the centimeter range [3, 4]. There have been a number of literatures at home and abroad on the calibration method of the single-element antenna phase center error of the navigation receiver, which has been deeply studied and systematically summarized [5]. However, now more and more receivers adopt multi-element array antennas, and there are few studies on the phase center of receiver array antennas. Array antennas have great advantages in anti-interference situations, but due to changes in the phase distribution of the antenna array, especially after using the anti-interference adaptive algorithm, the phase center will change greatly, causing the navigation positioning accuracy to deteriorate [6, 7]. Therefore, it is of great practical significance to study the phase center distribution law of array antennas.

For the phase center calculation of the array antenna: In the literature [8, 9] taking the phased array antenna as an example, several methods for calculating the phase center of the phased array antenna are introduced. Some methods have reference value for calculating the phase center of the satellite navigation array antenna. However, the number, structure, and signal processing modes of the two antenna elements are usually quite different, and the navigation array antenna needs to be considered separately when calculating its phase center. In [10, 11], taking the linear array as an example, the relationship between the phase pattern of the array antenna and the phase shift of the phase center is derived, and the exact value of the phase center is calculated by the least squares method. The algorithm only considers the linear array antenna. The two principal planes whose directions are perpendicular to each other and the phase center of any section are not analyzed in detail.

When using an anti-jamming antenna array, in addition to the anti-interference ability, the phase center changes need to be considered. Especially before the anti-interference processing, the antenna array should have a phase center that is as stable as possible. In this paper, a common star-shaped antenna array is taken as an example. Different array elements are selected, and the antenna array and its phase center model are established. The phase center errors are compared to provide a reference basis for selecting which antenna array to choose.

## 2. Star-shaped Array Antenna and its Phase Center Model

### 2.1 Star-shaped Array Antenna Model

As shown in Fig. 1, in the Cartesian coordinate system, the direction of any signal source  $s$  can be expressed as  $(\theta, \varphi)$ , where  $\theta$  is the zenith angle,  $\varphi$  is the azimuth angle (the subsequent expression is similar), and the phase value of the  $O$  point affected by  $s$  is recorded as  $\psi_0(\theta, \varphi)$  at a certain moment, and the corresponding unit direction vector is  $\mathbf{r} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$ . The wavelength of the navigation satellite signal is  $\lambda$  and the wave number is  $k = 2\pi/\lambda$ . For the  $N$ -element array antenna, the sitting of the  $i$ -th array element  $F_i$  is marked as  $(x_i, y_i, z_i)$  ( $i=1, 2, \dots, N$ ), and the phase of the signal  $s$  to  $F_i$  at a certain moment is  $\psi_0(\theta, \varphi) - k\mathbf{OF}_i \cdot \mathbf{r}$ . Under the condition of  $\psi_i(\theta, \varphi) = k\mathbf{OF}_i \cdot \mathbf{r}$ , the steering vector of the signal  $s$  is [12]

$$\mathbf{a}(\theta, \varphi) = [e^{-j\psi_1(\theta, \varphi)}, e^{-j\psi_2(\theta, \varphi)}, \dots, e^{-j\psi_N(\theta, \varphi)}]^T \quad (1)$$

Where T represents a matrix transpose.

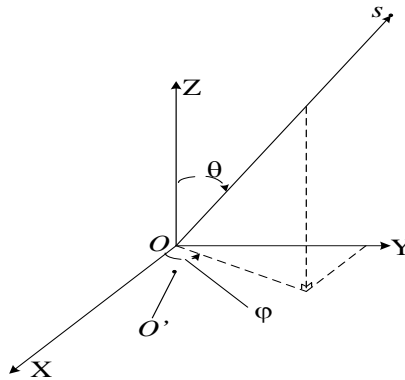


Figure 1. Schematic diagram of antenna geometry

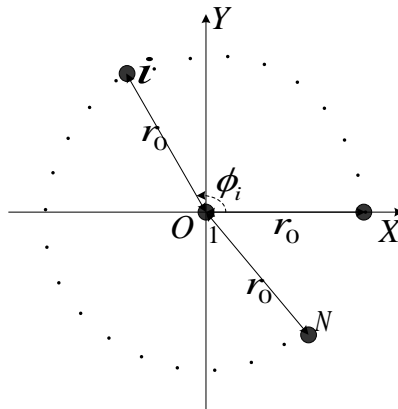


Figure 2. Star-shaped array antenna structure model

Considering star-shaped array antenna composed of  $N$  elements, as shown in Figure 2, one array element is at the origin, the remaining array elements are at distance  $r_0$  from the origin, and the angle from the x-axis clockwise to the  $i$ -array element ( $i = 2, 3, \dots, N$ ) is  $\phi_i = \frac{2\pi(i-2)}{N-1}$ . To study the phase change of the array antenna, it is necessary to consider the direction function of the antenna. According to the reciprocity theorem of the antenna [13], the receiver characteristics of the receiver antenna are consistent with the radiation characteristics, and the antenna of the array element at the origin can be set far away from the direction  $(\theta, \varphi)$ . The field radiation direction function is [14]:

$$f_0(\theta, \varphi) = A_0(\theta, \varphi) e^{j\psi_0(\theta, \varphi)} \quad (2)$$

Where  $A_0(\theta, \varphi)$  is the real amplitude value. Regardless the mutual coupling of the array elements, for the array antenna shown in Fig. 2, the weight of each array element is equal, and the weight matrix is  $\mathbf{w} = \underbrace{[1, 1, \dots, 1]^T}_{N \times 1}$ , and the far-field radiation direction function is

$$f(\theta, \varphi) = \sum_{i=1}^N w_i A_0(\theta, \varphi) e^{j[\psi_0(\theta, \varphi) - k\mathbf{O}\mathbf{F}_i \cdot \mathbf{r}]}, \text{ that is,}$$

$$f(\theta, \varphi) = f_0(\theta, \varphi) \mathbf{w}^T \mathbf{a}(\theta, \varphi) \quad (3)$$

If equation (3) is expressed as  $f(\theta, \varphi) = A_m e^{j\psi(\theta, \varphi)}$ , where  $A_m$  is a real function, the phase direction function of the array antenna is  $\psi(\theta, \varphi)$ .

## 2.2 Array Antenna Phase Center Error Model

The geometric center of the antenna is usually used as the antenna reference point. For the antenna array shown in Fig. 2, the phase value of the far-field signal  $s$  relative to the reference point  $O$  is obtained from the phase direction function, and is denoted as  $\psi(\theta, \varphi)$ . If the  $O'(x_0, y_0, z_0)$  is the phase center of the antenna array, the phase difference between the points  $O'$  and  $O$  at the influence of  $s$  is  $k\mathbf{O}\mathbf{O}' \cdot \mathbf{r}$ , and the phase difference between  $s$  and  $O'$  is  $\psi(\theta, \varphi) - k\mathbf{O}\mathbf{O}' \cdot \mathbf{r}$ . According to the definition of the phase center, the phase difference of the key area of the radiation (in the region around the sphere where  $s$  is located with  $O'$  as the center) relative to  $O'$  is a constant  $C$ , then [8]

$$C = \psi(\theta, \varphi) - k\mathbf{O}\mathbf{O}' \cdot \mathbf{r} \quad (4)$$

For a phase center, set the phase of  $s$  and its adjacent far-field spherical point to  $\psi(\theta_i, \varphi_i)$  ( $i = 1, 2, \dots, n$  and  $n \geq 4$ ), which can be considered to satisfy the condition of equation (4).

$$k(x_0 \sin \theta_i \cos \varphi_i + y_0 \sin \theta_i \sin \varphi_i + z_0 \cos \theta_i) + C = \psi(\theta_i, \varphi_i) \quad (5)$$

$$\text{Assuming } \mathbf{A} = \begin{bmatrix} k \sin \theta_1 \cos \varphi_1 & k \sin \theta_1 \sin \varphi_1 & k \cos \theta_1 & 1 \\ k \sin \theta_2 \cos \varphi_2 & k \sin \theta_2 \sin \varphi_2 & k \cos \theta_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ C \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \psi(\theta_1, \varphi_1) \\ \psi(\theta_2, \varphi_2) \\ \vdots \end{bmatrix}, \quad \mathbf{B} \text{ can}$$

be calculated from the equation in Section 2.1, then  $\mathbf{A}\mathbf{X} = \mathbf{B}$ , the least squares solution is  $\mathbf{X} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{B}$ , and the phase center is  $(x_0, y_0, z_0)$ . In this paper, the distance from the phase center of the star-shaped array antenna to the reference point is taken as the phase center error.

### 3. Simulation calculation

Under the condition of no interference, the L1 frequency of the GPS is 1575.42MHz as the carrier frequency of the navigation satellite, and the phase center error of the different elements is simulated by MATLAB. In order to balance the size of the antenna array and reduce the mutual coupling between the elements, the array distance  $r_0$  takes the half-wavelength of the carrier [15]. The mutual coupling between the elements is not considered in this simulation. The calculation technique for the phase center of each array element is relatively mature. If the influence is considered, it only needs to be corrected by its geometric relationship. In this paper, for the simplified calculation, the geometric center of each array element is used as the phase center of each array element.

#### 3.1 Phase center error caused by different azimuth angles

Under different numbers of array elements, when the zenith angle  $\theta$  of the satellite signal is  $15^\circ$ , the phase center error caused by the different azimuth angles is analyzed, as shown in Fig. 3.

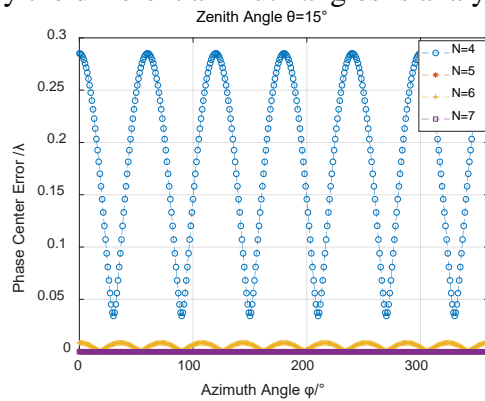


Figure 3. Phase center error under azimuthal variation (N is the number of elements)

When other conditions are consistent, it can be seen from FIG. 3 that if the azimuth of the satellite changes, the phase center error of the array antenna has a periodic characteristic, and the increase of the number of array elements causes the period of the phase center error to be shortened. With the same azimuth angle, the array antenna error composed of odd array elements is close to 0, and the phase center error of the array antenna composed of even array elements is smaller. When the satellite zenith angle takes other angles, it still conforms to the above rules. The reason for this phenomenon is that the number of the peripheral elements in the odd array antenna is even and symmetrically distributed, which can better offset the error variation caused by the geometric distribution.

#### 3.2 Phase center error caused by different zenith angles

Under different numbers of array elements, when the azimuth angle  $\phi$  of the satellite signal is  $190^\circ$ , the phase center error caused by the different zenith angles is analyzed, as shown in Fig. 4.

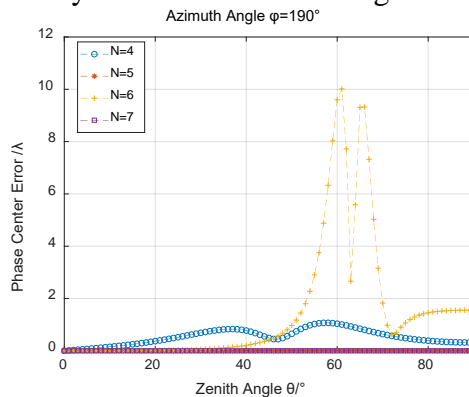


Figure 4. Phase center error under zenith angle variation (N is the number of elements)

When other conditions are consistent, as can be seen from FIG. 4 that if the zenith angle of the satellite increases, the phase center error of the array antenna fluctuates, and if the zenith angle is close to  $0^\circ$ , the phase center error of the array antenna is close to 0, and it does not gradually increase with the increase of the zenith angle. When there is the same zenith angle, the phase center error of array antenna composed of odd array elements is still close to 0, and its error is smaller than that of array antenna composed of even array elements.

#### 4. Conclusion

Based on the star-shaped array antenna model, this paper compares the phase center errors of antenna arrays composed of different numbers of array elements without interference. The analysis shows that, when there is no interference, the phase center of the antenna array composed of odd array elements coincides with the geometric center of the antenna array. The phase center of the antenna array composed of even array elements is related to the satellite signal direction: the phase center error is periodic when the azimuth is increasing or decreasing; the phase center error is close to 0 when the zenith angle is close to zero. In order to improve the positioning accuracy as much as possible, the stability of phase center is an important factor to be considered when selecting star-shaped anti-interference array antenna composed of different array elements. The method and conclusions of this paper can provide reference for it.

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